

SOLUTION:

1. THE SUM OF EVEN INTEGERS:

The sum of the even numbers between 1 and k is $79 \cdot 80$, where k is an odd number, then $k = ?$

- (A) 79
- (B) 80
- (C) 81
- (D) 157
- (E) 159

The solution given in the file was 79, which is **not correct**. Also the problem was solved with AP formula thus was long and used the theory rarely tested in GMAT. Here is my solution and my notes about AP. Some may be useful:

The number of terms in this set would be: $n = (k-1)/2$ (as k is odd)

Last term: $k-1$

Average would be first term + last term $/ 2 = (2 + k - 1) / 2 = (k + 1) / 2$

Also average: sum / number of terms $= 79 \cdot 80 / ((k-1)/2) = 158 \cdot 80 / (k-1)$
 $(k+1)/2 = 158 \cdot 80 / (k-1) \rightarrow (k-1)(k+1) = 158 \cdot 160 \rightarrow k = 159$

Answer E.

MY NOTES ABOUT AP:

ARITHMETIC PROGRESSION

Sequence a_1, a_2, \dots, a_n , so that $a(n) = a(n-1) + d$ (constant)

n th term $a_n = a_1 + d(n-1)$

$S_n = n(a_1 + a_n)/2$ or $S_n = n(2a_1 + d(n-1))/2$

Special cases:

I. $1 + 2 + \dots + n = n(n+1)/2$ (Sum of n first integers)

II. $1 + 3 + 5 + \dots$ (n times) $= n^2$ (Sum of n first odd numbers). n th term $= 2n - 1$

III. $2 + 4 + 6 + \dots$ (n times) $= n(n+1)$ (Sum of n first even numbers) n th term $= 2n$

SOLUTION WITH THE AP FORMULA:

Sequence of even numbers

First term $a = 2$, common difference $d = 2$ since even number

Sum to first n numbers of AP:

$S_n = n(a_1 + a_n)/2 = n(2 + 2(n-1))/2 = n(n+1) = 79 \cdot 80$

$n = 79$ (odd)

Number of terms $n = (k-1)/2 = 79$ $k = 159$

OR

Sum of n even numbers $n(n+1) = 79 \cdot 80$

$n = 79$

$k = 2n + 1 = 159$

SOLUTION:

2. THE PRICE OF BUSHEL:

The price of a bushel of corn is currently \$3.20, and the price of a peck of wheat is \$5.80. The price of corn is increasing at a constant rate of $5x$ cents per day while the price of wheat is decreasing at a constant rate of $\sqrt{2} \cdot x - x$ cents per day. What is the approximate price when a bushel of corn costs the same amount as a peck of wheat?

- (A) \$4.50
- (B) \$5.10
- (C) \$5.30
- (D) \$5.50
- (E) \$5.60

Note that we are not asked in how many days prices will cost the same.

Let y be the # of days when these two bushels will have the same price.

First let's simplify the formula given for the rate of decrease of the price of wheat: $\sqrt{2} \cdot x - x = 1.41x - x = 0.41x$, this means that the price of wheat decreases by $0.41x$ cents per day, in y days it'll decrease by $0.41xy$ cents;

As price of corn increases $5x$ cents per day, in y days it'll will increase by $5xy$ cents;

Set the equation: $320 + 5xy = 580 - 0.41xy$, solve for $xy \rightarrow xy = 48$;

The cost of a bushel of corn in y days (the # of days when these two bushels will have the same price) will be $320 + 5xy = 320 + 5 \cdot 48 = 560$ or \$5.6.

Answer: E.

SOLUTION:

3. LEAP YEAR:

How many randomly assembled people are needed to have a better than 50% probability that at least 1 of them was born in a leap year?

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Probability of a randomly selected person NOT to be born in a leap year = $3/4$

Among 2 people, probability that none of them was born in a leap = $3/4 \cdot 3/4 = 9/16$. The probability at least one born in leap = $1 - 9/16 = 7/16 < 1/2$

So, we are looking for such n (# of people), when $1 - (3/4)^n > 1/2$
 $n=3 \rightarrow 1 - 27/64 = 37/64 > 1/2$

Thus min 3 people are needed.

Answer: C.

4. ADDITION PROBLEM:

$AB + CD = AAA$, where AB and CD are two-digit numbers and AAA is a three digit number; A, B, C , and D are distinct positive integers. In the addition problem above, what is the value of C ?

- (A) 1
- (B) 3
- (C) 7
- (D) 9
- (E) Cannot be determined

AB and CD are two digit integers, their sum can give us only one three digit integer of a kind of AAA it's 111.

So, $A=1$. $1B + CD = 111$

C can not be less than 9, because no to digit integer with first digit 1 (mean that it's < 20) can be added to two digit integer less than 90 to have the sum 111 (if $CD < 90$ meaning $C < 9$ $CD + 1B < 111$).

$C=9$

Answer: D.

SOLUTION:

5. RACE:

A and B ran, at their respective constant rates, a race of 480 m. In the first heat, A gives B a head start of 48 m and beats him by 1/10th of a minute. In the second heat, A gives B a head start of 144 m and is beaten by 1/30th of a minute. What is B's speed in m/s?

- (A) 12
- (B) 14
- (C) 16
- (D) 18
- (E) 20

Let x be the speed of B.

Write the equation:

$(480-48)/x$ (time of B for first heat) - 6 (seconds, time B lost to A first heat) = *TIME OF A (in both heats A runs with constant rate, so the time for first and second heats are the same)* = $(480-144)/x$ (time of B for second heat) + 2 (seconds, time B won to A second heat)

$$(480-48)/x - 6 = (480-144)/x + 2$$

$$x = 12$$

Answer: A.

SOLUTION:

6. PROBABILITY OF DRAWING:

A bag contains 3 red, 4 black and 2 white balls. What is the probability of drawing a red and a white ball in two successive draws, each ball being put back after it is drawn?

- (A) 2/27
- (B) 1/9
- (C) 1/3
- (D) 4/27
- (E) 2/9

This is with replacement case (and was solved incorrectly by some of you):

$$P = 2 * \frac{3}{9} * \frac{2}{9} = \frac{4}{27}$$

We are multiplying by 2 as there are two possible winning scenarios RW and WR.

Answer: D.

SOLUTION:

7. THE DISTANCE BETWEEN THE CIRCLE AND THE LINE:

What is the least possible distance between a point on the circle $x^2 + y^2 = 1$ and a point on the line $y = 3/4x - 3$?

- A) 1.4
- B) sqrt (2)
- C) 1.7
- D) sqrt (3)
- E) 2.0

This is tough:

First note that min distance from the circle to the line would be: length of perpendicular from the origin to the line (as the circle is centered at the origin) - the radius of a circle (which is 1).

Now we can do this by finding the equation of a line perpendicular to given line $y = \frac{3}{4}x - 3$ (we know it should cross origin and cross given line, so we can write the formula of it), then find the cross point of these lines and then the distance between the origin and this point. But it's very lengthy way.

There is another, shorter one. Though I've never seen any GMAT question requiring the formula used in it.

We know the formula to calculate the distance between two points (x_1, y_1) and (x_2, y_2)

: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ BUT there is a formula to calculate the distance between the point (in our case origin) and the line:

DISTANCE BETWEEN THE LINE AND POINT:

Line: $ay + bx + c = 0$, point (x_1, y_1)

$$d = \frac{|ay_1 + bx_1 + c|}{\sqrt{a^2 + b^2}}$$

DISTANCE BETWEEN THE LINE AND ORIGIN:

As origin is $(0, 0)$ -->

$$d = \frac{|c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|3|}{\sqrt{1^2 + (\frac{3}{4})^2}} = \frac{12}{5} = 2.4$$

So in our case it would be:

$$\text{So the shortest distance would be: } 2.4 - 1(\text{radius}) = 1.4$$

Answer: A.

OR ANOTHER APPROACH:

In an x-y Cartesian coordinate system, the circle with center (a, b) and radius r is the set of all points (x, y) such that:

$$(x - a)^2 + (y - b)^2 = r^2$$

If the circle is centered at the origin (0, 0), then the equation simplifies to:

$$x^2 + y^2 = r^2$$

So, the circle represented by the equation $x^2 + y^2 = 1$ is centered at the origin and has the radius of $r = \sqrt{1} = 1$.

Then note that min distance from the circle to the line would be: length of perpendicular from the origin to the line (as the circle is centered at the origin) - the radius of a circle (which is 1).

So we should find the length of perpendicular, or the height of the right triangle formed by the X and Y axis and the

line $y = \frac{3}{4}x - 3$.

The legs would be the value of x for y=0 (x intercept) --> y=0, x=4 --> $leg_1 = 4$.

and the value of y for x=0 (y intercept) --> x=0, y=-3 --> $leg_2 = 3$.

So we have the right triangle with legs 4 and 3 and hypotenuse 5. What is the height of this triangle (perpendicular from right angle to the hypotenuse)? As perpendicular to the hypotenuse will always divide the triangle into two triangles with the same properties as

$$\text{the original triangle: } \frac{\text{height}}{leg_1} = \frac{leg_2}{\text{hypotenuse}} \dots \frac{\text{height}}{3} = \frac{4}{5} \dots \text{height} = 2.4.$$

$$\text{Distance} = \text{height} - \text{radius} = 2.4 - 1 = 1.4$$

Answer: A.

SOLUTION OF 8-10

8. THE AVERAGE TEMPERATURE:

The average of temperatures at noontime from Monday to Friday is 50; the lowest one is 45, what is the possible maximum range of the temperatures?

- A. 20
- B. 25
- C. 40
- D. 45
- E. 75

Average=50, Sum of temperatures= $50 \times 5 = 250$

As the min temperature is 45, max would be $250 - 4 \times 45 = 70 \rightarrow$ The range= $70(\text{max}) - 45(\text{min}) = 25$

Answer: B.

9. PROBABILITY OF INTEGER BEING DIVISIBLE BY 8:

If n is an integer from 1 to 96 (inclusive), what is the probability for $n(n+1)(n+2)$ being divisible by 8?

- A. 25%
- B. 50%
- C. 62.5%
- D. 72.5%
- E. 75%

$$N = n(n+1)(n+2)$$

N is divisible by 8 in two cases:

When n is even:

No of even numbers (between 1 and 96)=48

AND

When $n+1$ is divisible by 8. $\rightarrow n=8p-1 \rightarrow 8p-1 \leq 96 \rightarrow p=12.3 \rightarrow 12$ such numbers

$$\text{Total} = 48 + 12 = 60$$

$$\text{Probability} = 60/96 = 0.625$$

Answer: C

10. SUM OF INTEGERS:

If the sum of five consecutive positive integers is A , then the sum of the next five consecutive integers in terms of A is:

- A. $A+1$ inquiry
- B. $A+5$
- C. $A+25$
- D. $2A$
- E. $5A$

Sum= A , next 5 consecutive will gain additional $5 \times 5 = 25$, so sum of the next five consecutive integers in terms of A is: $A+25$

Answer: C.